PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Doug Hensley, Douglas B. West**
with the collaboration of Itshak Borosh, Paul Bracken, Ezra A. Brown, Randall Dougherty, Tamás Erdélyi, Zachary Franco, Christian Friesen, Ira M. Gessel, László Lipták, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Steven J. Miller, Mohamed Omar, Richard Pfiefer, Dave Renfro, Cecil C. Rousseau, Leonard Smiley, Kenneth Stolarsky, Richard Stong, Walter Stromquist, Daniel Ullman, Charles Vanden Eynden, and Fuzhen Zhang.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal, nor posted to the internet before the due date for solutions. Submitted solutions should arrive before Nov 30, 2016. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11915. *Proposed by Mark E. Kidwell and Mark D. Meyerson, U.S. Naval Academy, Annapolis, MD.* Given four points A, B, C, and D in order on a line in Euclidean space, under what conditions will there be a point P off the line such that the angles $\angle APB$, $\angle BPC$, and $\angle CPD$ have equal measure?

11916. *Proposed by Hideyuki Ohtsuka, Saitama, Japan, and Roberto Tauraso, Universitá di Roma "Tor Vergata," Rome, Italy.* Show that if n, r, and s are positive integers, then

$$\binom{n+r}{n}\sum_{k=0}^{s-1}\binom{r+k}{r-1}\binom{n+k}{n} = \binom{n+s}{n}\sum_{k=0}^{r-1}\binom{s+k}{s-1}\binom{n+k}{n}.$$

11917. *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.* Let A be a 2×2 matrix with integer entries and both eigenvalues less than 1 in absolute value. Prove that $\log(I-A)$ has integer entries if and only if $A^2=0$. (Here $\log(I-X) = -X - X^2/2 - X^3/3 - \cdots$ when that sum converges.)

11918. *Proposed by Le Van Phu Cuong, College of Education, Hue University, Hue City, Vietnam.* Let f be n times continuously differentiable on $[0,1]$, with $f(1/2)=0$ and $f^{(i)}(1/2)=0$ when i is even and at most n. Prove

$$\left(\int_0^1 f(x)\,dx\right)^2 \le \frac{1}{(2n+1)2^{2n}(n!)^2}\int_0^1 f^{(n)}(x))^2\,dx.$$

11919. *Proposed by Arkady Alt, San Jose, CA.* For positive integers m and k with $k\ge 2$, prove

$$\sum_{i_1=1}^{n}\cdots\sum_{i_k=1}^{n}(\min\{i_1,\dots,i_k\})^m = \sum_{i=1}^{m}(-1)^{m-i}\binom{m}{i}((n+1)^i-n^i)\sum_{j=1}^{n}j^{k+m-i}.$$

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